

# C.U.SHAH UNIVERSITY

## Winter Examination-2015

**Subject Name: Engineering Mathematics-II**

**Subject Code: 4TE02EMT2**

**Semester: II**

**Time: 10.30 To 1:30**

**Branch: B.Tech(All)**

**Date: 19/11/2015**

**Marks: 70**

**Instructions:**

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

**Q-1 Attempt the following questions:**

**(14)**

a)  $\int_{-\pi/2}^{\pi/2} \sin^7 x \, dx = \underline{\hspace{2cm}}$

- (a) 0      (b) 1      (c)  $\frac{\pi}{2}$       (d)  $\frac{1}{2}$

b)  $\int_0^{\pi/2} \cos^4 x \, dx = \underline{\hspace{2cm}}$

- (a) 0      (b) 1      (c)  $\frac{3\pi}{16}$       (d)  $\frac{8\pi}{3}$

c)  $\int_0^1 \int_0^x dy \, dx = \underline{\hspace{2cm}}$

- (a)  $\frac{1}{2}$       (b) -1      (c) 0      (d) y

d) The value of  $\int_{-\pi}^{\pi} \sin mx \sin nx \, dx$  for  $m \neq \pm n$  is

- (a) 0      (b)  $\pi$       (c)  $\frac{\pi}{2}$       (d)  $2\pi$

e)  $\beta(1,1) = \underline{\hspace{2cm}}$

- (a) 0      (b) 1      (c)  $\sqrt{\pi}$       (d)  $\pi$

f)  $\Gamma(n)\Gamma\left(n + \frac{1}{2}\right) = \underline{\hspace{2cm}}$

- (a)  $\frac{\sqrt{\pi} \Gamma(2n)}{2^{2n-1}}$       (b)  $\frac{\sqrt{\pi} \Gamma(2n)}{2^{2n}}$       (c)  $\frac{\sqrt{\pi} \Gamma(n)}{2^{2n-1}}$       (d)  $\frac{\sqrt{\pi} \Gamma(n)}{2^{2n}}$



- g)** If the two tangents at the point are real and coincident, the double point is called \_\_\_\_\_.
- (a) a node (b) a cusp (c) a conjugate point (d) none of these
- h)** The curve passes through the origin, if the equation does not contain \_\_\_\_\_
- (a) terms in  $x$  (b) terms in  $y$  (c) constant term (d) none of these
- i)** Length of curve for  $y = f(x)$  is defined by
- (a)  $\int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$  (b)  $\int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$
- (c)  $\int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dx$  (d)  $\int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dx$
- j)**  $\int_0^1 \int_1^2 \int_0^3 dx dy dz =$  \_\_\_\_\_
- (a) 1 (b) -3 (c)  $\frac{1}{3}$  (d) 3
- k)** The degree of the differential equation  $\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^3\right]^{\frac{2}{3}}$  is
- (a) 1 (b) 2 (c) 3 (d) 6
- l)** The order of the differential equation  $\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}} = \left[y + 5\left(\frac{dy}{dx}\right)\right]^{\frac{1}{2}}$  is
- (a) 1 (b)  $\frac{1}{2}$  (c)  $\frac{3}{2}$  (d) 2
- m)** The  $p$ -series  $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots + \frac{1}{n^p} + \dots$  is convergent for
- (a)  $p < 1$  (b)  $p > 1$  (c)  $p = 1$  (d) none of these
- n)** The series  $1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} + \dots$  is
- (a) convergent (b) divergent (c) oscillatory (d) none of these

**Attempt any four questions from Q-2 to Q-8**

**Q-2 Attempt all questions**

- a)** Find the volume common to the cylinder  $x^2 + y^2 = a^2$  and  $x^2 + z^2 = a^2$ . **(05)**
- b)** Evaluate:  $\int_0^{\pi} x \sin^7 x \cos^4 x dx$  **(05)**



- c) Prove that (i)  $\operatorname{erfc}(x) + \operatorname{erfc}(-x) = 2$   
(ii)  $\operatorname{erf}(x) + \operatorname{erfc}(x) = 1$  (04)

**Q-3 Attempt all questions**

a) Evaluate:  $\int_0^1 \left(x \log \frac{1}{x}\right)^n dx$  (05)

b) Test for the convergence the series  $\sum_{n=1}^{\infty} \frac{[(n+1)x]^n}{n^{n+1}}$  (05)

c) Solve:  $\frac{dy}{dx} + y \tan x = \sin 2x, y(0) = 1$  (04)

**Q-4 Attempt all questions**

a) Test for convergence the series  $2 + \frac{3}{2}x + \frac{4}{3}x^2 + \frac{5}{4}x^3 + \dots$  (05)

b) Trace the curve  $r = a(1 + \cos \theta)$ . (05)

c) Prove that (i)  $n\beta(m+1, n) = m\beta(m, n+1)$   
(ii)  $\beta(m, n) = \beta(m, n+1) + \beta(m+1, n)$  (04)

**Q-5 Attempt all questions**

a) Evaluate:  $\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2 + y^2 + z^2) dz dy dx$  (05)

b) Solve:  $\frac{dy}{dx} = 2y \tan x + y^2 \tan^2 x$  (05)

c) Test for convergence the series  $4 - 1 + \frac{1}{4} - \frac{1}{16} + \dots$  and if it is convergent then also find its sum. (04)

**Q-6 Attempt all questions**

a) Evaluate  $\iint_R xy \, dy \, dx$ , where  $R$  is the positive quadrant of the circle  $x^2 + y^2 = a^2$  (05)

b) Derive Reduction formula for  $\int_0^{\frac{\pi}{2}} \cos^n x \, dx, n \geq 2$ . (05)

c) Find the orthogonal trajectories of the family of parabola  $y = ax^2$ . (04)



**Q-7 Attempt all questions**

a) Change the order of integration and evaluate  $\int_0^a \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} dx dy$ . (05)

b) A circuit containing a resistance  $R$ , an inductance  $L$  in series is acted on by periodic electromotive force  $E \sin \omega t$ . If  $i = 0$  when  $t = 0$ , show that the current at any time  $t$  is  $i(t) = \frac{E}{\sqrt{R^2 + L^2 \omega^2}} \left\{ \sin(\omega t - \phi) + e^{-\frac{Rt}{L}} \sin \phi \right\}$ , where  $\phi = \tan^{-1} \left( \frac{L\omega}{R} \right)$  (05)

c) Solve:  $(y^2 e^{xy^2} + 4x^3) dx + (2xy e^{xy^2} - 3y^2) dy = 0$  (04)

**Q-8 Attempt all questions**

a) Evaluate:  $\int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{2ax - x^2} \sqrt{a^2 - x^2}}$  (05)

b) Trace the curve  $y^2(2a - x) = x^3$ . (05)

c) Find the perimeter of the cardioid  $r = a(1 + \cos \theta)$  (04)

